JW-003-001543 Seat No. ________

B. Sc. (Sem. V) (CBCS) Examination

October - 2019

Mathematical Statistics : S-502

(Old Course)

Faculty Code : 003
Subject Code : 001543

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions :

(1) Q. 1 carry 20 marks.
(2) Q. 2 and Q. 3 carry 25 marks each.
(3) Student can use their own scientific calculator.

1 Filling the blanks and short questions : (each 1 mark) **20**

(1) ______ is a characteristic function of Poisson distribution.

(2) ______ is a characteristic function of Standard Normal distribution.

(3) ______ is a characteristic function of Geometric distribution.

(4) ______ is a characteristic function of chi-square distribution.

(5) ______ is a moment generating function of $\gamma(\alpha, p)$.

(6) ______ is a moment generating function of chi-square distribution.

(7) For Normal distribution $\mu_{2n} =$ ______.

(8) For Normal distribution $\mu_4 = k_4 + 3k_2^2$ is _____.

(9) If two independent variates $X_1 \sim N\left(\mu_1, \sigma_1^2\right)$ and $X_2 \sim N\left(\mu_2, \sigma_2^2\right)$ then $X_1 + X_2$ is distributed as _____.

(10) If two independent variates $X_1 \sim \gamma(n_1)$ and $X_2 \sim \gamma(n_2)$ then $\frac{X_1}{X_1 + X_2}$ is distributed as _____.

(11) If two independent variates $X_1 \sim \Lambda\left(\mu_1, \sigma_1^2\right)$ and $X_2 \sim \Lambda\left(\mu_2, \sigma_2^2\right)$ then $X_1 \cdot X_2$ is distributed as _____.

(12) Weibull distribution has application in _____.

(13) If χ_1^2 and χ_2^2 are two independent chi-square variates with d.f. n_1 and n_2, respectively, then the distribution of $\frac{\chi_1^2}{\chi_2^2}$ is ______.

(14) Pearson's coefficient of skewness for chi-square distribution curve is ______.

(15) $t-$distribution curve in respect of tails is always _____.

(16) Given a joint bivariate normal distribution of X, Y as $BVN\left(\mu_X, \mu_Y, \sigma_X^2, \sigma_Y^2, \rho\right)$, the marginal distribution $f_Y(y) =$_______.

(17) If the variablexs (X, Y) follow BVN (1, 2, 4, 9, 0.5) distribution, the conditional distribution of $(X / Y = 5)$ has mean _____ and variance ______.

(18) A measure of linear association of a variable say, X_1 with a number of other variables $X_2, X_3, X_4, \ldots\ldots\ldots, X_k$ is known as ________.

(19) The range of multiple correlation coefficient R is ____.

(20) Partial correlation coefficient is the simple correlation between ______ variables.

2 (a) Write the answer any three : (each 2 marks) **6**

(1) Why characteristic function need ?

(2) If $u = \frac{x-a}{h}$, a and h being constants then $\varnothing_u(t) = e^{\left(-iat/h\right)} \varnothing_x\left(t/h\right)$.

(3) Define Weibul distribution.

(4) Define truncated distribution.

(5) Prove that $\sigma_{3.12}^2 = \dfrac{\sigma_3^2\left(1 - r_{12}^2 - r_{23}^2 - r_{13}^2 + 2r_{12}\, r_{23}\, r_{13}\right)}{\left(1 - r_{12}^2\right)}$.

(6) In trivariate distribution it is found that

$r_{12} = 0.77, r_{13} = 0.72$ and $r_{23} = 0.52$.

Find :

(i) $r_{12.3}$

(ii) $R_{1.23}$

(b) Write the answer any three : (each 3 marks) **9**

(1) Obtain probability density function for the characteristic function $\varnothing_X(t) = p\left(1 - qe^{it}\right)^{-1}$.

(2) Obtain mean and variance of Beta distribution of second kind.

(3) Define exponential distribution and obtain its moment generating function (MGF). From MGF obtain its mean and variance.

(4) Define truncated Poisson distribution and also obtain its mean and variance.

(5) Usual notation of multiple correlation and multiple regression, prove that $b_{12} = \dfrac{b_{12.3} + b_{13.2}\, b_{32.1}}{1 - b_{13.2}\, b_{31.2}}$.

(6) Usual notation of multiple correlation and multiple regression, prove that $\sigma_{1.23}^2 = \sigma_1^2\left(1 - r_{12}^2\right)\left(1 - r_{13.2}^2\right)$.

(c) Write the answer any two : (Each 5 marks) **10**

(1) State and prove that Chebchev's inequality.

(2) Derive F-distribution.

(3) If x and y are independent χ^2 variates with n_1 and n_2 degree of freedom respectively, then obtain distribution of $\dfrac{x}{x+y}$ and $x + y$.

(4) Obtain conditional distribution of y when x is given for bi-variate distribution.

(5) Usual notation of multiple correlation and multiple regression, prove that $R_{1.23}^2 = \dfrac{r_{12}^2 + r_{13}^2 - 2r_{12}\, r_{23}\, r_{13}}{1 - r_{23}^2}$.

3 (a) Write the answer any three : (each 2 marks) **6**

(1) Define Beta-I and beta-II distribution.

(2) Define log normal distribution when $y = \log_e(x-a)$.

(3) Obtain characteristic function of Poisson distibution with parameter λ.

(4) Obtain relation between t and F distribution.

(5) Usual notation of multiple correlation and multiple regression, prove that $R_{1.23}^2 = b_{12.3}\, r_{12} \frac{\sigma_2}{\sigma_1} + b_{13.2}\, r_{13} \frac{\sigma_3}{\sigma_1}$.

(6) Prove that $b_{12.3} = \frac{b_{12} - b_{13}\, b_{23}}{1 - b_{13}\, b_{23}}$.

(b) Write the answer any three : (each 3 marks) **9**

(1) Prove that $\mu_r' = (-i)^r \left[\frac{d^r}{dt^r} \varnothing_X(t) \right]_{t=0}$.

(2) Obtain moment generating function (MGF) of Normal distribution.

(3) Obtain mean and variance of uniform distribution.

(4) Obtain harmonic mean of $X \sim \gamma(\alpha, p)$.

(5) Usual notation of multiple correlation and multiple regression, prove that $r_{xy} + r_{yz} + r_{xz} \geq -\frac{3}{2}$.

(6) Usual notation of multiple correlation and multiple regression, prove that, if $r_{12} = r_{23} = r_{31} = r$ then $R_{1.23} = R_{2.31} = R_{3.12} = \frac{\sqrt{2}r}{\sqrt{1+r}}$.

(c) Write the answer any two : (each 5 marks) **10**

(1) Obtain moment generating function (MGF) and cumulate generating function (CGF) of Gamma distribution with parameters α and p. Also show that $\mu_4 = k_4 + 3k_2^2$.

(2) Derive Normal distribution.

(3) Derive $t-$distribtuion.

(4) Drive χ^2 distribution and show that $3\beta_1 - 2\beta_2 - 6 = 0$.

(5) Usual notation of multiple correaltion and multiple regression, prove that $r_{12.3} = \frac{r_{12} - r_{13}\, r_{23}}{\sqrt{\left(1 - r_{13}^2\right)\left(1 - r_{23}^2\right)}}$.
